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# Inventory Models with Variable Demand and Deterioration

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**Abstract:**
This paper deals with the inventory models of deteriorating products for which demand rate depends on stock-level as well as time and deterioration has been considered as linear function of time and constant.

**Key-words:** Demand, deterioration and stock level.

**1 Introduction:**
The classical EOQ model does not have involvement of stock level q or time t on demand rate. So many authors have produced in past, several inventory models, taking the dependence of stock level or time on demand rate. **Gupta & Vrat (**1988), **Mandal & Phaujdar** (1989) considered the stock dependent demand rate in their EOQ models. **Silver & Meal** (1973) developed an approximate solution procedure for the general case of a deterministic, time varying demand pattern. The classical no-shortage inventory problem for a linear trend in demand over a finite time-horizon was analytically solved by **Donaldson** (1977). **Ritchie** (1984) obtained an exact solution, having the simplicity of the EOQ formula, for **Donaldson** problem for linearly increasing demand. **Deb & Chaudhari** (1987) discussed the inventory replenishment policy for items having a deterministic demand pattern with a linear trend and shortages. They developed a heuristic to determine the decision rule and sizes of replenishment over a finite time horizon to keep the total cost minimum. **Murdeshwar** (1988) extended this work.
Deterioration is an important factor in the study of some inventory models. Deterioration is defined as decay, damage or spoilage of items such that it can not be used for its original purpose. Food items, drugs, photographic films, pharmaceuticals and radioactive substances are some examples of items in which deterioration can take place during the normal storage period of units. Therefore this loss must be taken into account in the analysis of the system. Inventory models for deteriorating or Perishable products subject to exogenous demands have been discussed by several authors. **Ghare & Schrader** (1963), **Covert & Philip** (1973) and **Mishra** (1975) discussed inventory models in which a constant or variable proportion of the on hand inventory deteriorates with time. **Mishra** (1995) presented an inventory model with instantaneous stock replenishment, demand rate depending on current stock level and a constant fraction of on hand inventory also deteriorates with time. He employed profit maximization technique to yield the optimal solution. Some functional relationships existing between the demand rate and current stock level were considered to establish the inventory model.

**Gupta & Vrat** (1988) developed an **i**nventory model for stock dependent consumption rate. **Hari Kishan & Mishra** (1995) discussed an **i**nventory model with stock dependent demand rate and constant rate of deterioration. **Hari Kishan & Mishra** (1997) developed an inventory model with exponential demand and constant rate of deterioration with shortage. Several other authors have developed several inventory models for stock dependent demand rate or time dependent demand rate during last decade for deteriorating products or non-deteriorating products.

In the above mentioned inventory models, the demand is either the function of stock level or time. But in some cases demand may depend on both stock-level as well as time. For example, the products showing a seasonal trend, has demand which depends on stock-level as well as time.
The present paper deals with the inventory models of deteriorating products for which demand rate depends on stock-level as well as time and deterioration has been considered as linear function of time and constant.

**2 Notations and Assumptions:**

The following notations are used in this paper:

(i) C is the unit cost price of the item.

(ii) $C_1$ the unit holding cost per unit of item.

(iii) S is the highest stock level.

(iv) A is the setup cost.

(v) T is the cycle time.

The following assumptions are used in this paper:

(i) In section-A, the demand rate is assumed of the forms

(a) r(q) = aq + b, a and b are constants

(b) r(q) = $q^2$ + c.

(ii) The deterioration rate $\theta$ is assumed to be constant i.e. a fractional part $\theta q$ deteriorates per unit time.

(iii) Shortages are not allowed.

(iv) In section-B, the demand rate is considered as the function of instantaneous stock level as well as time. It is assumed of the form

(c) $r(q,t) = \alpha + \beta q^{\chi} + \delta t^{\lambda} + \theta q$

(d) $r(q,t) = \alpha + \beta e^{q} + \chi e^{t} + \theta q$

(e) $r(q,t) = \alpha + \beta q^2 t^2 + \theta q$

**3 Mathematical Models:**
**Section –A:**

**Profit Maximization Technique:**
Here the EOQ model has been developed in the case where the replenishment of the stock is instantaneous, shortages are not allowed and the demand rate depends upon the current stock level as well as time. The total profit per unit time during time T, obtained by **Mandal & Phaujdar**(1989) is given by

$$Z(S) = \frac{pS - [A + CS - C_1 G(S)]}{F(S)}. \qquad \ldots(1)$$

where p denotes the unit selling price of the item, A the setup cost, C the unit cost price of the item, $C_1$ the unit holding cost per unit of item and S is the highest stock level.

Also
$$F(S) = \int_0^S \frac{dq}{r(q)} = T, \qquad \ldots(2)$$

and
$$G(S) = \int_0^S \frac{q\,dq}{r(q)}. \qquad \ldots(3)$$

The optimum value of S for the total maximum profit per unit time is a solution of $Z'(S) = 0$ provided $Z''(S) < 0$ for that value of S. Thus for optimal value of S, expression (1) implies

$$F(S)[(p-C) - C_1 G'(S)] = F'(S)[(p-C)S - A - C_1 G(S)], \qquad \ldots(4)$$

where prime denotes the derivative w.r.t. S. Equation (4) is in general a non-linear equation in S which can be solved, numerically, by Newton-Raphson method if the explicit form of r(q) is known. The optimal cycle length is given by $F(S^*)$ where S* is the optimal value of S.

Let $q(t)$ be the inventory level at any time t, $\theta qt$ be the deterioration and $tr(q)$ be the demand rate. Let T be the length of each cycle of inventory. Then the differential equation governing the system is given by

$$\frac{dq}{dt} + \theta qt = -tr(q)$$

or $$\frac{dq}{dt} = -(\theta q + r(q))t. \quad \text{…(5)}$$

Considering q(0)=S and q(T)=0, the length T of each cycle is calculated by using equation (3.5) as

$$\int_0^T tdt = \int_0^S \frac{dq}{\theta q + r}$$

or $$\frac{T^2}{2} = \int_0^S \frac{dq}{\theta q + r} = f(s), \text{ (say)} \quad \text{…(6)}$$

or $$T = \sqrt{2}\sqrt{f(s)} = F(S), \text{ (say)} \quad \text{…(7)}$$

Now we consider the following two forms of r(q):

(i) r(q) = aq + b, a and b are constants

(ii) r(q) = $q^2$ + c

**Case (i):** Let r(q) = aq + b. Then (6) gives

$$\frac{T^2}{2} = \int_0^S \frac{dq}{\theta q + aq + b}$$

$$= \int_0^S \frac{dq}{(a+\theta)q + b}$$

$$= \left[\frac{\log[(a+\theta)q + b]}{(a+\theta)}\right]_0^S$$

$$= \frac{1}{(a+\theta)}\log\left[\frac{(a+\theta)S + b}{b}\right]$$

or $$T = \sqrt{\frac{2}{(a+\theta)}\log\left[\frac{(a+\theta)S + b}{b}\right]} = F(S) \text{ (say)} \quad \text{…(8)}$$

Differentiating (8) w.r.t. S, we get

$$F'(S) = \frac{1}{[(a+\theta)S + b]}\sqrt{\frac{(a+\theta)}{2\log\left[\frac{(a+\theta)S + b}{b}\right]}} \quad \text{…(9)}$$

Also from (5), on integrating we have

$$\frac{\log[(a+\theta)q + b]}{(a+\theta)} = -\frac{t^2}{2} + B, \quad \text{…(10)}$$

where B is the constant of integration.

Applying the boundary conditions

$q(0)=S$ in (10), we get

$$B=\frac{\log[(a+\theta)S+b]}{(a+\theta)}$$

Therefore (3.10) reduces to

$$\frac{\log[(a+\theta)q+b]}{(a+\theta)}=-\frac{t^2}{2}+\frac{\log[(a+\theta)S+b]}{(a+\theta)}$$

or $$q=\left[S+\frac{b}{a+\theta}\right]e^{Zt^2/2}-\frac{b}{a+\theta}. \qquad \ldots(11)$$

Now total amount of inventory during the cycle [0,T] is given by

$$G(S)=\int_0^T q\,dt$$

$$=\int_0^T\left[\left[S+\frac{b}{a+\theta}\right]e^{-(a+\theta)t^2/2}-\frac{b}{a+\theta}\right]dt$$

$$=\int_0^T\left[\left(S+\frac{b}{a+\theta}\right)\left(1-\frac{(a+\theta)}{2}t^2\right)-\frac{b}{a+\theta}\right]dt$$

$$=\int_0^T\left[S\left(1-\frac{(a+\theta)}{2}t^2\right)-\frac{b}{2}t^2\right]dt$$

$$=\left[S\left(t-\frac{(a+\theta)}{6}t^3\right)-\frac{b}{6}t^3\right]_0^T$$

$$=S\left[T-\frac{(a+\theta)}{6}T^3\right]-\frac{bT^3}{6}$$

$$=ST-\frac{[S(a+\theta)+b]}{6}T^3. \qquad \ldots(12)$$

Substituting the value of T from (8) in the above equation (12), we get

$$G(S)=S\left[\frac{2}{(a+\theta)}\log\left[\frac{(a+\theta)S+b}{b}\right]\right]^{1/2}$$

$$-\frac{[S(a+\theta)+b]}{6}\left[\frac{2}{(a+\theta)}\log\left[\frac{(a+\theta)S+b}{b}\right]\right]^{3/2}. \qquad \ldots(13)$$

Differentiating the expression (13) w.r.t. S, we get

$$G'(S)=\left[\frac{1}{(a+\theta)}\log\left[\frac{(a+\theta)S+b}{b}\right]\right]\left[1-\frac{2}{3}\log\left[\frac{(a+\theta)S+b}{b}\right]\right]+\frac{S}{2}. \qquad \ldots(14)$$

Finally, for the total profit to be maximum the optimal condition (4) using the values of $F(S)$, $F'(S)$, $G(S)$ and $G'(S)$ from equations (8), (9), (13) and (14) respectively, implies

$$\sqrt{\frac{2}{(a+\theta)}\log\left[\frac{(a+\theta)S+b}{b}\right]}\,[(\text{p-C})$$

$$-C_1\left[\frac{1}{(a+\theta)}\log\left[\frac{(a+\theta)S+b}{b}\right]\right]\left[1-\frac{2}{3}\log\left[\frac{(a+\theta)S+b}{b}\right]\right]+\frac{S}{2}]$$

$$=\frac{1}{[(a+\theta)S+b]}\sqrt{\frac{(a+\theta)}{2\log\left[\frac{(a+\theta)S+b}{b}\right]}}\ [(p-C)S-A$$

$$-C_1S\left[\frac{2}{(a+\theta)}\log\left[\frac{(a+\theta)S+b}{b}\right]\right]^{1/2}$$

$$-C_1\frac{[S(a+\theta)+b]}{6}\left[\frac{2}{(a+\theta)}\log\left[\frac{(a+\theta)S+b}{b}\right]\right]^{3/2}\ ]$$

This is a non-linear equation in S which can be solved, numerically, by Newton-Raphson's method. The optimal cycle length is given by $F(S^*)$ where S* is the optimal value of S.

The condition for total average cost to be minimum is given by p=0 in the above expression.

**Case (ii):** Let r(q) = $q^2+c$. Then (6) gives

$$\frac{T^2}{2}=\int_0^s\frac{dq}{\theta q+q^2+c}$$

$$=\int_0^s\frac{dq}{q^2+\theta q+c}$$

$$=\int_0^s\frac{dq}{(q+\frac{\theta}{2})^2+\left(a-\frac{\theta^2}{4}\right)}$$

$$=\frac{1}{\sqrt{\left(a-\frac{\theta^2}{4}\right)}}\left[\tan^{-1}\frac{q+\frac{\theta}{2}}{\sqrt{\left(a-\frac{\theta^2}{4}\right)}}\right]_0^S$$

$$=\frac{1}{\sqrt{\left(a-\frac{\theta^2}{4}\right)}}\left[\tan^{-1}\frac{S\sqrt{\left(a-\frac{\theta^2}{4}\right)}}{\left(a-\frac{\theta^2}{4}\right)+\frac{\theta}{2}\left(S+\frac{\theta}{4}\right)}\right]$$

or $$T=\frac{\sqrt{2}}{\left(a-\frac{\theta^2}{4}\right)^{1/4}}\left[\tan^{-1}\frac{S\sqrt{\left(a-\frac{\theta^2}{4}\right)}}{\left(a-\frac{\theta^2}{4}\right)+\frac{\theta}{2}\left(S+\frac{\theta}{4}\right)}\right]^{1/2}=F_1(S),\ \text{(say)}.\qquad\ldots(15)$$

Differentiating (15) w.r.t. S, we get

$$F_1'(S)=\frac{\left(a-\frac{\theta^2}{4}\right)^{1/4}}{\sqrt{2(a+\theta S)+S^2}}\cdot\frac{1}{\sqrt{\tan^{-1}\frac{S\sqrt{a-\frac{\theta^2}{4}}}{(a+\theta S)}}}. \qquad \ldots(16)$$

Also from (5) on integrating, we have

$$\int\frac{dq}{q^2+\theta q+a}=-\frac{t^2}{2}+B_1,$$

or
$$\frac{1}{\sqrt{a-\frac{\theta^2}{4}}}\tan^{-1}\frac{\left(q+\frac{\theta}{2}\right)}{\sqrt{a-\frac{\theta^2}{4}}}=-\frac{t^2}{2}+B_1. \qquad \ldots(17)$$

Applying the boundary condition q(0) = S in (17), we get

$$B_1=\frac{1}{\sqrt{a-\frac{\theta^2}{4}}}\tan^{-1}\frac{\left(S+\frac{\theta}{2}\right)}{\sqrt{a-\frac{\theta^2}{4}}}.$$

Therefore, (17) reduces to

$$\frac{1}{\sqrt{a-\frac{\theta^2}{4}}}\tan^{-1}\left[\frac{q+\frac{\theta}{2}}{\sqrt{a-\frac{\theta^2}{4}}}\right]=-\frac{t^2}{2}+\frac{1}{\sqrt{a-\frac{\theta^2}{4}}}\tan^{-1}\left[\frac{\left(S+\frac{\theta}{2}\right)}{\sqrt{a-\frac{\theta^2}{4}}}\right]$$

or
$$q=-\frac{\theta}{2}+\sqrt{a-\frac{\theta^2}{4}}\tan\left[\tan^{-1}\left(\frac{S+\frac{\theta}{2}}{\sqrt{a-\frac{\theta^2}{4}}}\right)-\frac{t^2\sqrt{a-\frac{\theta^2}{4}}}{2}\right]. \qquad \ldots(18)$$

Now the total amount of inventory during the cycle [0,T] is given by

$$G_1(S)=\int_0^T q\,dt$$

$$\int_0^T\left[-\frac{\theta}{2}+\sqrt{\left(a-\frac{\theta^2}{4}\right)}\tan\left\{\tan^{-1}\left(\frac{S+\frac{\theta}{2}}{\sqrt{a-\frac{\theta^2}{4}}}\right)-\frac{t^2\sqrt{a-\frac{\theta^2}{4}}}{2}\right\}\right]dt. \qquad \ldots(19)$$

From (19) we can obtain the value of $G_1'(S)$.

Finally, for the total profit to be maximum the optimal condition (4) using the values of $F_1(S)$, $F_1'(S)$, $G_1(S)$ and $G_1'(S)$ from equations (15), (16) and (19) respectively, implies

$$\frac{\sqrt{2}}{\left(a-\frac{\theta^2}{4}\right)^{1/4}}\left[\tan^{-1}\frac{S\sqrt{\left(a-\frac{\theta^2}{4}\right)}}{\left(a-\frac{\theta^2}{4}\right)+\frac{\theta}{2}\left(S+\frac{\theta}{4}\right)}\right]^{1/2}[(p-S)-C_1G_1^{'}(S)]$$

$$=\frac{\left(a-\frac{\theta^2}{4}\right)^{1/4}}{\sqrt{2(a+\theta S)+S^2}}\cdot\frac{1}{\sqrt{\tan^{-1}\frac{S\sqrt{a-\frac{\theta^2}{4}}}{(a+\theta S)}}}[(p-C)S-A$$

$$-C_1\int_0^T\left[-\frac{\theta}{2}+\sqrt{\left(a-\frac{\theta^2}{4}\right)}\tan\left\{\tan^{-1}\left(\frac{S+\frac{\theta}{2}}{\sqrt{a-\frac{\theta^2}{4}}}\right)-\frac{t^2\sqrt{a-\frac{\theta^2}{4}}}{2}\right\}\right]dt\,. \quad \ldots(20)$$

This is a non-linear equation in S which can be solved, numerically, by Newton-Raphson's method. The optimal cycle length is given by $F(S^*)$ where S* is the optimal value of S.
The condition for total average cost to be minimum is given by p=0 in the above expression.

**Section – B:**
**Cost Minimization Technique:**
In this section, some bivariable demand rates have been considered for constant deterioration case. A cost minimization criterion is applied to derive the total average cost per unit time of the system. Functional forms of the demand rate are:

(i) $r(q,t)=\alpha+\beta q^{\gamma}+\delta t^{\lambda}+\theta q$

(ii) $r(q,t)=\alpha+\beta e^{q}+\delta e^{t}+\theta q$

(iii) $r(q,t)=\alpha+\beta q^{2}t^{2}+\theta q$

**Case (i):**
Let demand rate be $r(q,t)=\alpha+\beta q^{\gamma}+\delta t^{\lambda}+\theta q$. The total average cost per unit time is given by

$$K(q,t)=C(\alpha+\beta q^{\gamma}+\delta t^{\lambda}+\theta q)+\frac{A}{q}(\alpha+\beta q^{\gamma}+\delta t^{\lambda}+\theta q)+\frac{CC_1}{2}q\,. \quad \ldots(21)$$

Differentiating (21) partially w.r.t q, we have

$$\frac{\partial K}{\partial q}=C\beta\gamma q^{\gamma-1}+\theta C-\frac{A\alpha}{q^2}+A\beta(\gamma-1)q^{\gamma-2}-\frac{A\delta t^{\lambda}}{q^2}+\frac{CC_1}{2} \quad \ldots(22)$$

$$\frac{\partial^2 K}{\partial q^2}=C\beta\gamma(\gamma-1)q^{\chi-2}+\frac{2A\alpha}{q^3}+A\beta(\gamma-1)(\gamma-2)q^{\chi-3}+\frac{2A\delta t^{\lambda}}{q^3} \quad \ldots(23)$$

Differentiating (21) partially w.r.t t, we have

$$\frac{\partial K}{\partial t}=C\delta\lambda t^{\lambda-1}+\frac{A\delta\lambda t^{\lambda-1}}{q} \quad \ldots(24)$$

$$\frac{\partial^2 K}{\partial t^2}=C\delta\lambda(\lambda-1)t^{\lambda-2}+\frac{A\delta\lambda(\lambda-1)t^{\lambda-2}}{q}. \qquad \ldots(25)$$

Differentiating (22) w.r.t. t, we have

$$\frac{\partial^2 K}{\partial q\partial t}=-\frac{A\lambda\delta t^{\lambda-1}}{q^2}. \qquad \ldots(26)$$

For K(q.t) to be minimum, the necessary conditions is

$$\left(\frac{\partial^2 K}{\partial q^2}\right)\left(\frac{\partial^2 K}{\partial t^{-2}}\right)-\left(\frac{\partial^2 K}{\partial k\partial t}\right)^2>0. \qquad \ldots(27)$$

Total average cost K will be minimum if $\frac{\partial^2 K}{\partial t^2}>0, \frac{\partial^2 K}{\partial q^2}>0$ and $\left(\frac{\partial^2 K}{\partial q^2}\right)\left(\frac{\partial^2 K}{\partial t^2}\right)-\left(\frac{\partial^2 K}{\partial q\partial t}\right)^2>0$ at $q=q^*$ and $t=t^*$ given by $\frac{\partial K}{\partial q}=0$ and $\frac{\partial K}{\partial t}=0$ provided $\alpha,\beta,\gamma$ and $\delta>0$ and $\gamma\geq 2$.

**Case (ii):**

Let the demand rate be $r(q,t)=\alpha+\beta e^q+\delta e^t+\theta q$ then total average cost per unit time is given by

$$K(q,t)=C(\alpha+\beta e^q+\delta e^t+\theta q)+\frac{A}{q}(\alpha+\beta e^q+\delta e^t)+\frac{CC_1}{2}q.$$

$$=C\alpha+\frac{CC_1A}{2}+C\theta q+\frac{A\alpha}{q}+C\beta e^q+C\delta e^t+\frac{A\beta}{q}e^q+\frac{A\delta}{q}e^t. \qquad \ldots(28)$$

Differentiating (28) partially w.r.t q, we have

$$\frac{\partial K}{\partial q}=c\theta-\frac{A\alpha}{q^2}+c\beta e^q-\frac{A\beta}{q^2}e^q+\frac{A\beta e^q}{q}-\frac{A\delta e^t}{q^2} \qquad \ldots(29)$$

$$\frac{\partial^2 K}{\partial q^2}=\frac{2A\alpha}{q^3}+C\beta e^q+\frac{2A\beta}{q^3}e^q-\frac{2A\beta}{q^2}e^q$$
$$+\frac{A\beta}{q}e^q+\frac{2A\delta}{q^3}e^q. \qquad \ldots(30)$$

Differentiating (28) partially w.r.t. t, we get

$$\frac{\partial K}{\partial t}=C\delta e^t+\frac{A\delta}{q}e^t \qquad \ldots(31)$$

$$\frac{\partial^2 K}{\partial t^2}=C\delta e^t+\frac{A\delta}{q}e^t. \qquad \ldots(32)$$

Differentiating (29) partially w.r.t. t, we get

$$\frac{\partial^2 k}{\partial t\partial q}=-\frac{A\delta}{q^2}e^t. \qquad \ldots(33)$$

For K(q,t) to be minimum, the necessary condition is

$$\frac{\partial K}{\partial q}=0 \text{ and } \frac{\partial K}{\partial t}=0.$$

or $$C\theta-\frac{A\alpha}{q^2}+C\beta e^q-\frac{A\beta}{q^2}e^q+\frac{A\beta e^q}{q}-\frac{A\delta e^t}{q^2}=0$$

and $$C\delta e^{t} + \frac{A\delta}{q} e^{t} = 0. \quad \ldots(34)$$

The total average cost K will be minimum if $\frac{\partial^2 K}{\partial t^2} > 0, \frac{\partial^2 K}{\partial q^2} > 0$ and $\left(\frac{\partial^2 K}{\partial q^2}\right)\left(\frac{\partial^2 K}{\partial t^2}\right) - \left(\frac{\partial^2 K}{\partial q \partial t}\right)^2 > 0$ at q=q* and t=t* given by $\frac{\partial K}{\partial q} = 0$ and $\frac{\partial K}{\partial t} = 0$.

**Case (iii):**

Let the demand rate be $r(q,t) = \alpha + \beta q^2 t^2 + \theta q$ then the total average cost per unit time is given by

$$K = (\alpha + \beta q^2 t^2 + \theta q)C + \frac{A}{q}(\alpha + \beta q^2 t^2 + \theta q) + \frac{CC_1}{2}$$

$$= C\alpha + C\beta q^2 t^2 + C\theta q + \frac{A}{q}\alpha + A\beta q t^2 + A\theta + \frac{CC_1}{2}. \quad \ldots(35)$$

Differentiating (35) partially w.r.t. q, we get

$$\frac{\partial K}{\partial q} = 2C\beta q t^2 + C\theta - \frac{A}{q^2}\alpha + A\beta t^2 \quad \ldots(36)$$

$$\frac{\partial^2 K}{\partial q^2} = 2C\beta t^2 + C\theta + \frac{2A}{q^3}\alpha. \quad \ldots(37)$$

Differentiating (35) partially w.r.t. t, we get

$$\frac{\partial K}{\partial t} = 2C\beta q^2 t + 2A\beta q t \quad \ldots(38)$$

$$\frac{\partial^2 K}{\partial t^2} = 2C\beta q^2 + 2A\beta q. \quad \ldots(39)$$

Differentiating (38) partially w.r.t. q, we get

$$\frac{\partial^2 K}{\partial t \partial q} = 4C\beta q t + 2A\beta t. \quad \ldots(40)$$

For K(q,t) to be minimum, the necessary condition is

$$\frac{\partial K}{\partial q} = 0 \text{ and } \frac{\partial K}{\partial t} = 0.$$

or $$2C\beta q t^2 + C\theta - \frac{A}{q^2}\alpha + A\beta t^2 = 0 \quad \ldots(41)$$

and $$2C\beta q^2 t + 2A\beta q t = 0. \quad \ldots(42)$$

The total average cost K will be minimum if $\frac{\partial^2 K}{\partial t^2} > 0, \frac{\partial^2 K}{\partial q^2} > 0$ and $\left(\frac{\partial^2 K}{\partial q^2}\right)\left(\frac{\partial^2 K}{\partial t^2}\right) - \left(\frac{\partial^2 K}{\partial q \partial t}\right)^2 > 0$ at q=q* and t=t* given by (41) and (42).

 

## 4 CONCLUSIONS:

In section-A profit maximization technique is used to estimate the optimal values of the stock level and the cycle time to maximize the total average profit while in section-B cost minimization technique is used to determine minimum average cost by determining the optimal values of the stock level and the time of the system respectively. Several demand rates are considered to develop the models. In section-B, the demand rate is assumed as the functions of instantaneous stock level as well as time. This is the more realistic situation.

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